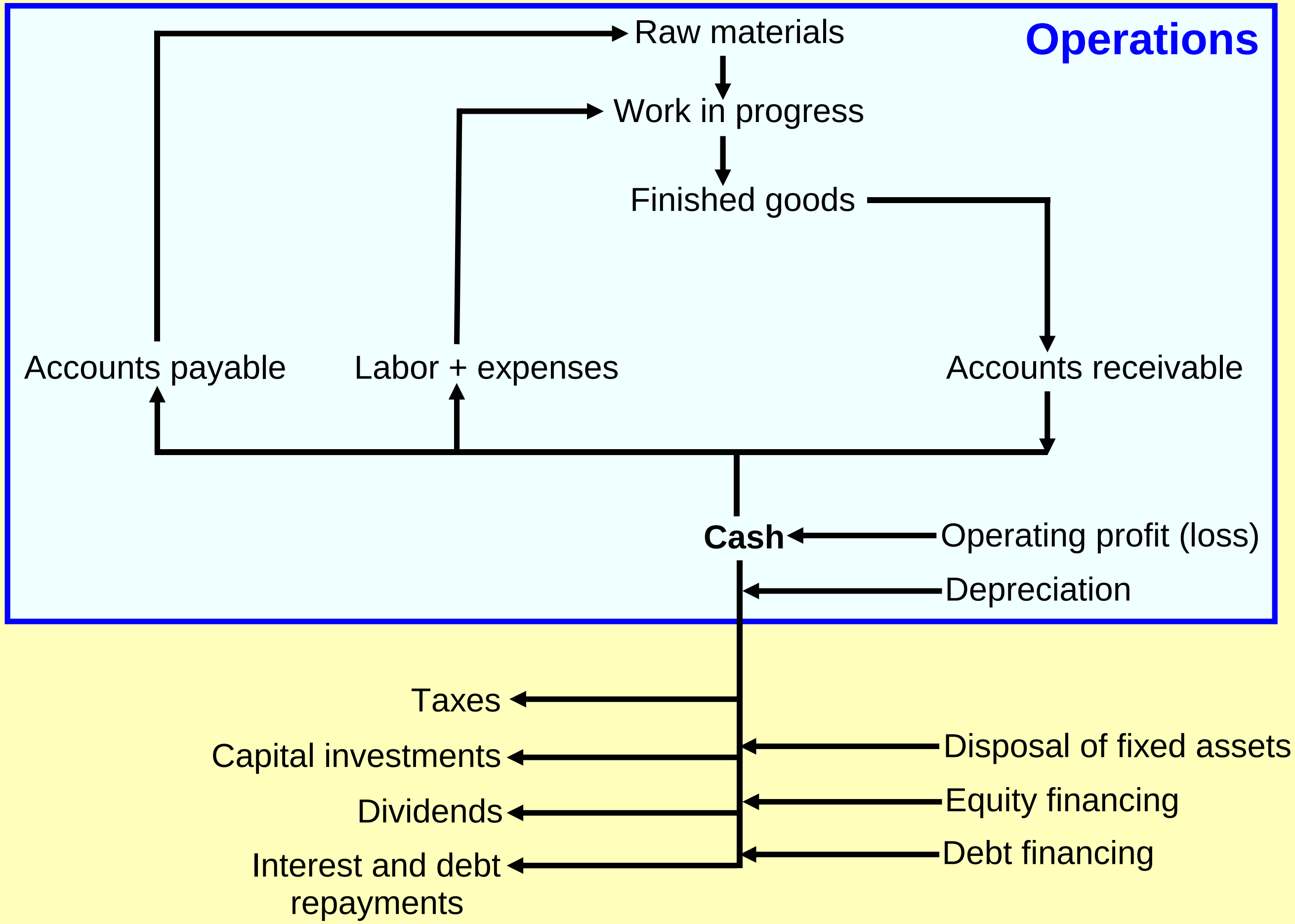
M591

Technology exploration project

http://cnfolio.com/IndexRiskAnalysis

System view of the cash flow cycle



Variance and Standard Deviation

- In the area of investment analysis, risk assessment must also include differences in expected values and the downside or loss potentials of the alternative investments.
- Consequently, the standard deviation can best be described as a measure of the "goodness" or confidence one can place in a best guess estimate (mean value) of the outcome of a random variable.
  - As applied to investment returns, the expected value may be the best guess of future returns.
  - If the standard deviation of returns is large, the best guess still may not be a very good guess.
- The standard deviation is a measure of the total risk of an asset or a portfolio.
  - Includes both systematic and unsystematic risk.
  - Captures the total variability in the asset's or portfolio's return, whatever the sources of that variability.

The **variance** measures the fluctuation of the observations around their mean. The larger the value of variance, the greater the fluctuation. The population variance is given by:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)^2$$

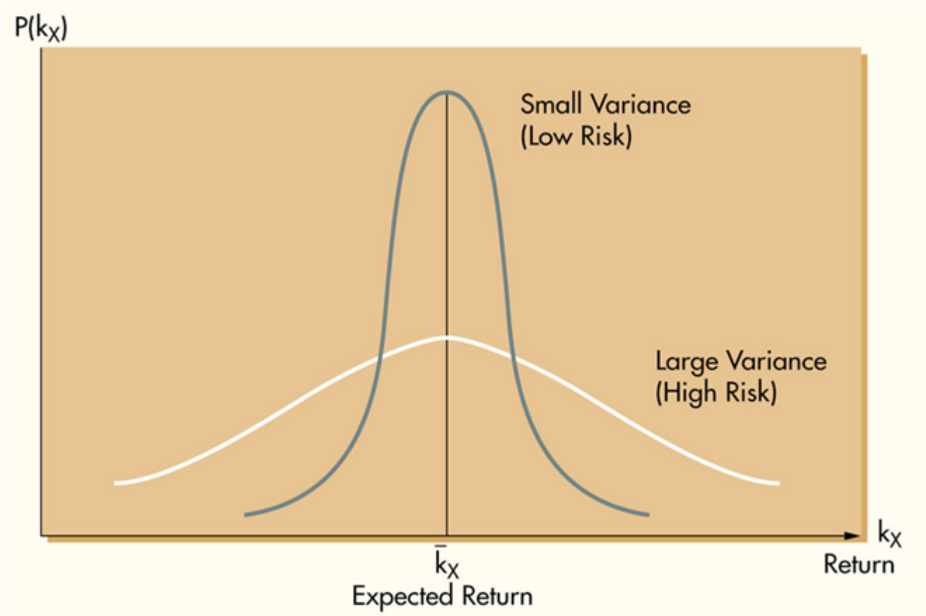
Where μ is the population mean and *n* represents the size of the population. The sample variance is given by:

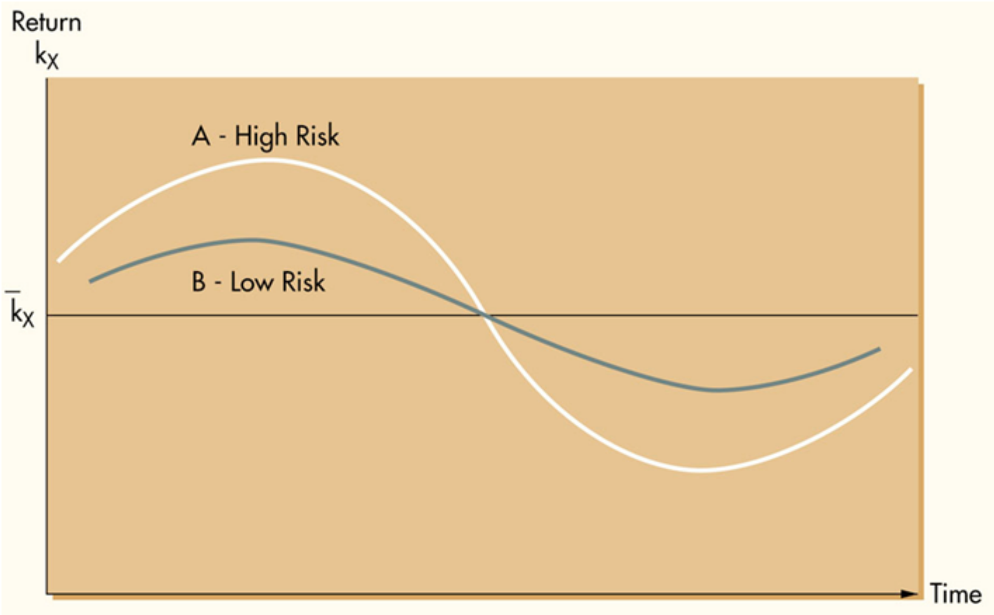
$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

Where *X* is the sample mean and *n* is the number of observations in the sample. In most applications, the sample variance is calculated rather than the population variance because calculation of the latter is possible only when every value in the population is known (i.e. the population variance has no practical application for empirical analyses). Dividing by *(n-1)* instead of by *n* (which may seem more logical) is done to make the sample variance correspond to σ. Division by n can be proven to produce an s-value that underestimates the population variance. Since our analyses are based on sample data with no complete understanding of the population, the sample variance from now on will be referred to as simply variance.

The standard deviation also measures the variability of observations around the mean. It is defined as the square root of the variance. The standard deviation will as a consequence have the same unit as the observation and is in a way easier to interpret. In financial terms, variability measured as standard deviation equals risk and the notion of risk has a very central place in the financial theory. From the definition of variance above follows that the standard deviation is given by:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)^2}$$


P(kX)
Small Variance
(Low Risk)
Large Variance
(High Risk)
kX
Return
kX
Expected Return


Return
k_X
A - High Risk
B - Low Risk
$\bar{k}_X$
Time

- Suppose we invest fraction w of our wealth in asset *A* and fraction (1–w) in asset *B*.
  - Suppose we have $100 in asset A and $300 in asset B, then total value of our portfolio is $400 and w is 0.25 (or 25%) and (1–w) is 0.75 (75%).
- What are the expected return and variance of our portfolio?
- The return on our portfolio is

$$R_P = wR_A + (1-w)R_B$$

Most investors do not keep all of their money invested in just one asset; instead, they hold collections of assets called **portfolios.** The fraction of the total portfolio invested in an individual asset is called the asset's **portfolio weight,** w_i. The **expected return on a portfolio,** $\hat{r}_p$, is the weighted average of the expected returns on the individual assets:

$$\text{Expected return on a portfolio} = \hat{r}_p = w_1\hat{r}_1 + w_2\hat{r}_2 + \cdots + w_n\hat{r}_n = \sum_{i=1}^{n} w_i\hat{r}_i \quad (29\text{-}11)$$

Here the $\hat{r}_i$ values are the expected returns on the individual assets.

The variance and standard deviation of a portfolio depend not only on the variances and weights of the individual assets in the portfolio, but also on the correlation between the individual assets. The **correlation coefficient** between two assets i and j, ρ_{ij}, can range from −1.0 to +1.0. If the correlation coefficient is greater than 0, the assets are said to be **positively correlated,** while if the correlation coefficient is negative, they are **negatively correlated.**[10] Returns on positively correlated assets tend to move up and down together, while returns on negatively correlated assets tend to move in opposite directions. For a two-asset portfolio with assets 1 and 2, the portfolio standard deviation, σ_p, is calculated as follows:

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2}} \qquad (29\text{-}12)$$

Here $w_2 = (1 - w_1)$, and $\rho_{1,2}$ is the correlation coefficient between assets 1 and 2. Notice that if w_1 and w_2 are both positive, as they must be unless one asset is sold short, then the lower the value of $\rho_{1,2}$, the lower the value of σ_p. This is an important concept: Combining assets that have low correlations results in a portfolio with a low risk. For example, suppose the correlation between two assets is negative, so when the return on one asset falls, then that on the other asset will generally rise. The positive and negative returns will tend to cancel each other out, leaving the portfolio with very little risk. Even if the assets are not negatively correlated, but have a correlation coefficient less than 1.0, say 0.5, combining them will still be beneficial, because when the return on one asset falls dramatically, that on the other asset will probably not fall as much, and it might even rise. Thus, the returns will tend to balance each other out, lowering the total risk of the portfolio.

The total risk (variance) of a portfolio of investments can be computed as:

$$\sigma_p^2 = \sum_{j=1}^{n} \sum_{i=1}^{n} x_i x_j \rho_{ij} \sigma_i \sigma_j$$

where the *xi yi* are the weights of each investment, the *ρ's* are the correlations between two investments, while the σ's are the standard deviations of each investment. The risk of additional investments is only relevant to the degree they correlate with the investments already in the portfolio. If *n* is large, the second term dominates the first one, making the individual variances completely unimportant.

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n = 2

j = 1, i = 1 $\quad x_1 x_1 \rho_{1,1} \sigma_1 \sigma_1 = x_1^2 \sigma_1^2$

j = 1, i = 2 $\quad x_2 x_1 \rho_{2,1} \sigma_2 \sigma_1 = x_1 x_2 \rho_{1,2} \sigma_1 \sigma_2$

j = 2, i = 1 $\quad x_1 x_2 \rho_{1,2} \sigma_1 \sigma_2 = x_1 x_2 \rho_{1,2} \sigma_1 \sigma_2$

(the two middle terms combine to) $2 x_1 x_2 \rho_{1,2} \sigma_1 \sigma_2$

j = 2, i = 2 $\quad x_2 x_2 \sigma_{2,2} \sigma_2 \sigma_2 = x_2^2 \sigma_2^2$

- Covariance is a measure of the degree to which two variables move in a systematic or predictable way, either positively or negatively.
  - If two variables move in perfect lockstep, up and down, they exhibit *perfect* positive covariance.
  - If two variables move in perfect lockstep, but in opposite directions, they exhibit *perfect* negative covariance.
  - If two variables are completely independent, showing no systematic relationship, their covariance is zero.

Formula for Covariance

$$\text{Cov}(X, Y) = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{n - 1} \quad \textbf{(3.5)}$$

Scatterplots that rise from lower left to upper right will tend to have positive covariance and correlation. Those that fall from upper left to lower right will tend to have negative covariance and correlation.

You probably will never have to use this formula directly—Excel has a built-in COVAR function that does it for you—but the formula does indicate what covariance is all about. It is essentially an average of products of deviations from means. If X and Y vary in the *same* direction, then when X is above (or below) its mean, Y will also tend to be above (or below) its mean. In either case, the product of deviations will be positive—a positive times a positive or a negative times a negative—so the covariance will be positive. The opposite is true when X and Y vary in *opposite* directions. Then the covariance will be negative.

The limitation of covariance as a descriptive measure is that it is affected by the *units* in which X and Y are measured. For example, we can inflate the covariance by a factor of 1000 simply by measuring X in dollars rather than in thousands of dollars. The correlation, denoted by Corr(X, Y), remedies this problem. It is a *unitless* quantity defined by equation (3.6), where Stdev(X) and Stdev(Y) denote the standard deviations of X and Y. Again, you'll probably never have to use this formula for calculations—Excel does it for you with the built-in CORREL function—but it does show that to produce a unitless quantity, we need to divide the covariance by the product of the standard deviations.

Formula for Correlation

$$\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\text{Stdev}(X) \times \text{Stdev}(Y)} \quad \textbf{(3.6)}$$

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$$\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\text{Stdev}(X) \times \text{Stdev}(Y)} \quad \textbf{(3.6)}$$

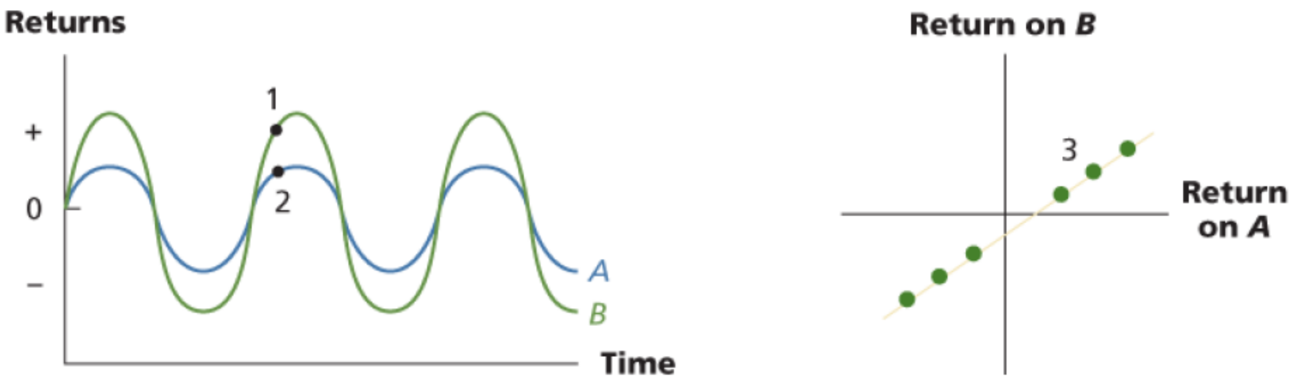
Statisticians use the symbol r for the correlation based on sample data. If they want to denote the correlation based on the entire population, they use the symbol ρ (rho).

The correlation is unaffected by the units of measurement of the two variables, and it is *always* between -1 and $+1$. The closer it is to either of these two extremes, the closer the points in a scatterplot are to some straight line, either in the negative or positive direction. On the other hand, if the correlation is close to 0, then the scatterplot is typically a "cloud" of points with no apparent relationship. However, it is also possible that the points are close to a *curve* and have a correlation close to 0. This is because correlation is relevant only for measuring *linear* relationships.

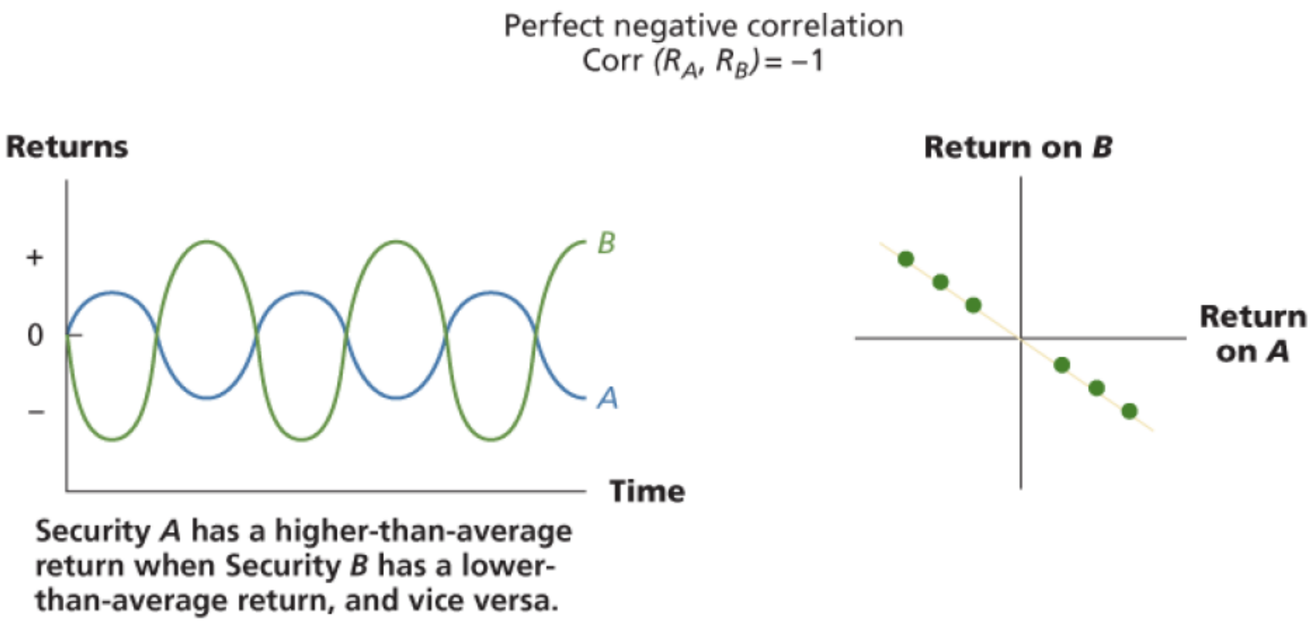
- The correlation between a variable and itself is always 1.
- The correlation between X and Y is the same as the correlation between Y and X. Therefore, it is sufficient to list the correlations below (or above) the diagonal in the table. (The same is true for covariances.) StatTools provides these options.
- The covariance between a variable and itself is the *variance* of that variable.

- It is difficult to interpret the magnitudes of the covariances. These depend on the fact that the data are measured in dollars rather than, say, thousands of dollars. It is much easier to interpret the magnitudes of the correlations because they are scaled to be between -1 and $+1$.

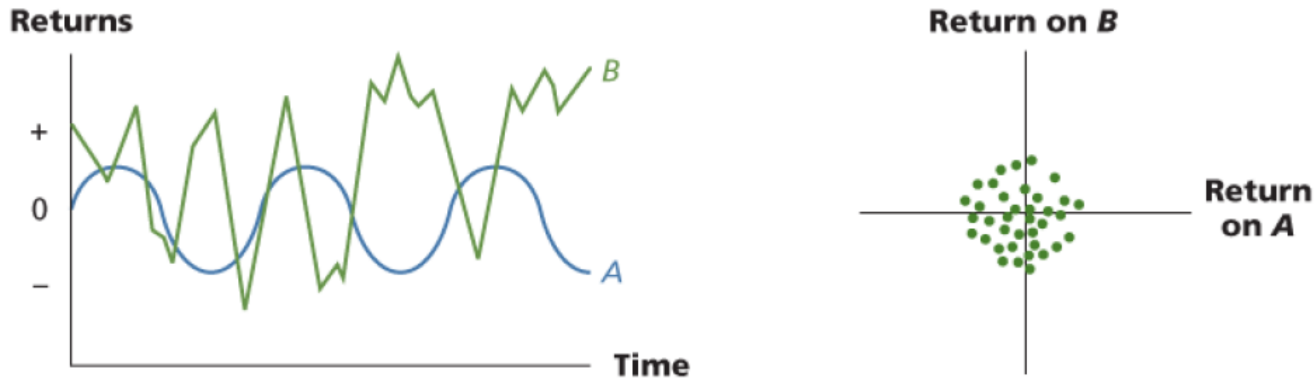
Perfect positive correlation
Corr $(R_A, R_B)=1$



Both the return on Security *A* and the return on Security *B* are higher than average at the same time. Both the return on Security *A* and the return on Security *B* are lower than average at the same time.


Perfect negative correlation
Corr $(R_A, R_B) = -1$
Returns
+
0
–
B
A
Time
Return on B
Return on A
Security A has a higher-than-average return when Security B has a lower-than-average return, and vice versa.

Zero correlation
Corr $(R_A, R_B) = 0$



The return on Security *A* is completely unrelated to the return on Security *B*.

- If two securities are perfectly negatively correlated, splitting one's investment equally between the two securities completely eliminates all variability.
- There is no risk reduction advantage to adding perfectly positively correlated assets to one's portfolio.
- In the real world, instances of either perfectly negatively correlated or perfectly positively correlated securities are extremely rare.
  - Investors do not need negative correlations between securities for them to benefit by adding securities to their portfolio.

- As long as a security added to a portfolio is not perfectly positively correlated with the existing portfolio, the addition of the security will reduce the portfolio's risk as measured by variance or standard deviation.
  - This concept is a central principle of modern portfolio and asset allocation theory.
- As long as there are any classes of assets whose returns are not perfectly correlated with investors' current portfolios, these investors can further reduce the risk of their portfolios by adding securities from those asset classes.

mean-variance analysis

To illustrate, suppose that in August 2006, an analyst estimates the following identical expected returns and standard deviations for Microsoft and General Electric:

	Expected Return, $\hat{r}$	Standard Deviation, σ
Microsoft	13%	30%
General Electric	13	30

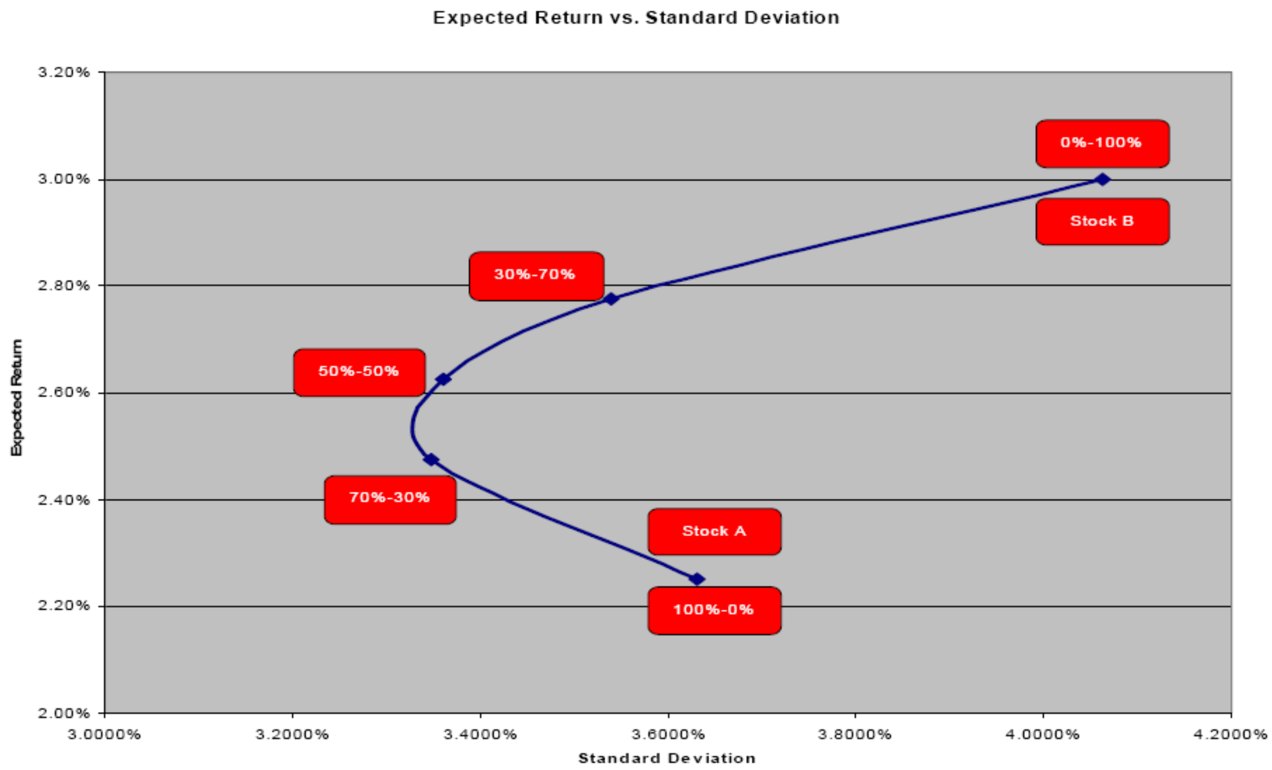
Suppose further that the correlation coefficient between Microsoft and GE is $\rho_{M,GE} = 0.4$. Now if you have \$100,000 invested in Microsoft, you will have a one-asset portfolio with an expected return of 13 percent and a standard deviation of 30 percent. Next, suppose you sell half of your Microsoft and buy GE, forming a two-asset portfolio with \$50,000 in Microsoft and \$50,000 in GE. (Ignore brokerage costs and taxes.) The expected return on this new portfolio will be the same 13.0 percent:

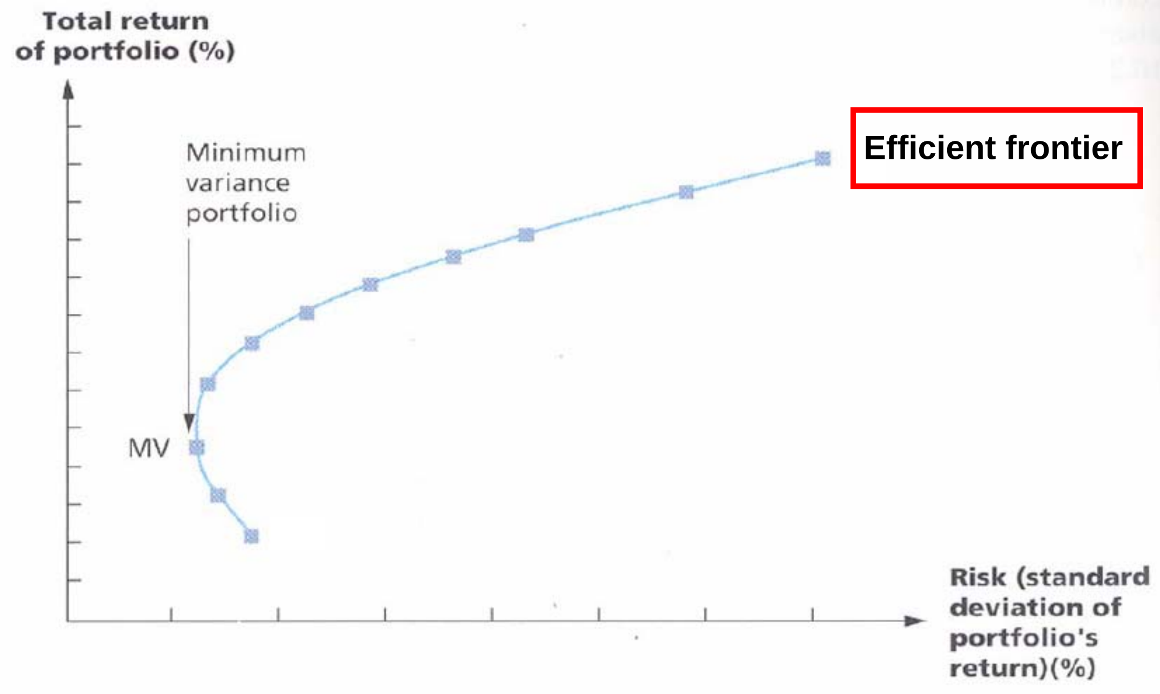
Since the new portfolio's expected return is the same as before, what's the point of the change? The answer, of course, is that diversification reduces risk. As noted above, the correlation between the two companies is 0.4, so the portfolio's standard deviation is found to be 25.1 percent:

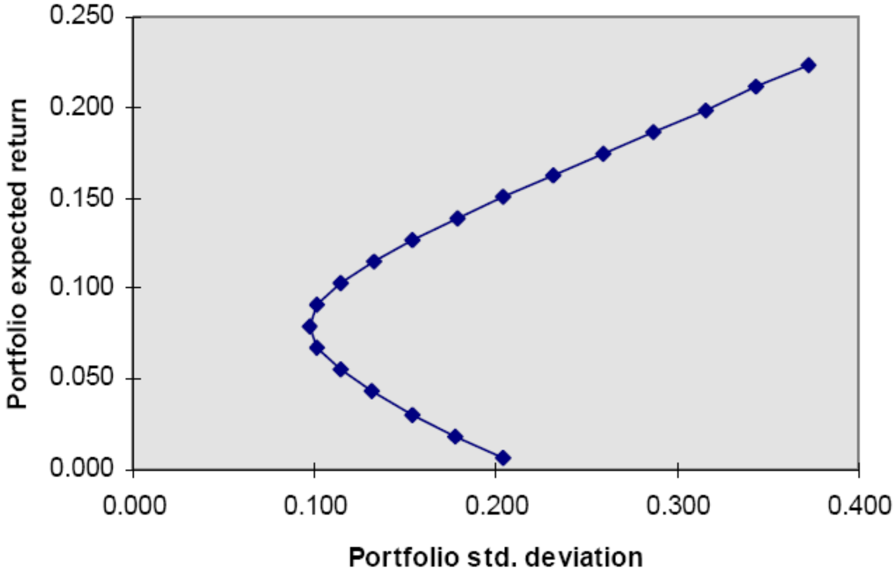
$$\begin{aligned}\sigma_p &= \sqrt{w_M^2\sigma_M^2 + w_{GE}^2\sigma_{GE}^2 + 2w_M w_{GE}\sigma_M\sigma_{GE}\rho_{M,GE}} \\ &= \sqrt{(0.5)^2(0.3)^2 + (0.5)^2(0.3)^2 + 2(0.5)(0.5)(0.3)(0.3)(0.4)} \\ &= \sqrt{0.0630} \\ &= 0.251 = 25.1\%\end{aligned}$$

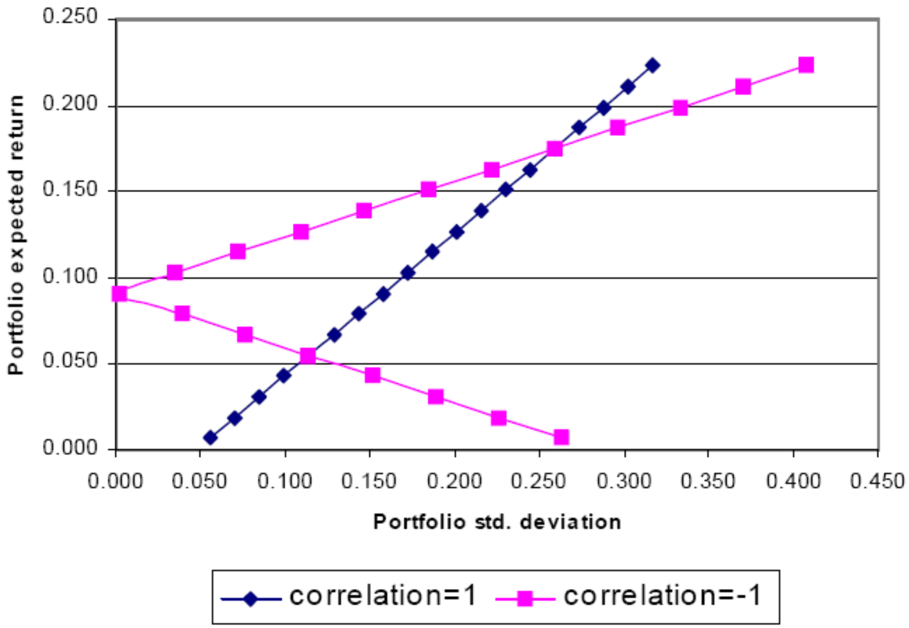
	Return A	Return B
Expected Return	2.25%	3.00%
Variance	0.001319	0.001650
St. Dev	3.6315%	4.0620%
Covariance	0.000775	
Correlation	0.525386	

- What are the expected return and variance on a portfolio of 50% stock A and 50% stock B?
- What are the expected return and variance on a portfolio of 70% stock A and 30% stock B?
- What are the expected return and variance on a portfolio of 30% stock A and 70% stock B?


Expected Return vs. Standard Deviation
Expected Return
Standard Deviation
3.20%
3.00%
2.80%
2.60%
2.40%
2.20%
2.00%
3.0000%
3.2000%
3.4000%
3.6000%
3.8000%
4.0000%
4.2000%
0%-100%
Stock B
30%-70%
50%-50%
70%-30%
Stock A
100%-0%


Total return
of portfolio (%)
Minimum
variance
portfolio
MV
Efficient frontier
Risk (standard
deviation of
portfolio's
return)(%)


0.250
0.200
0.150
0.100
0.050
0.000
Portfolio expected return
0.000
0.100
0.200
0.300
0.400
Portfolio std. deviation


0.250
0.200
0.150
0.100
0.050
0.000
Portfolio expected return
0.000
0.050
0.100
0.150
0.200
0.250
0.300
0.350
0.400
0.450
Portfolio std. deviation
correlation=1
correlation=-1

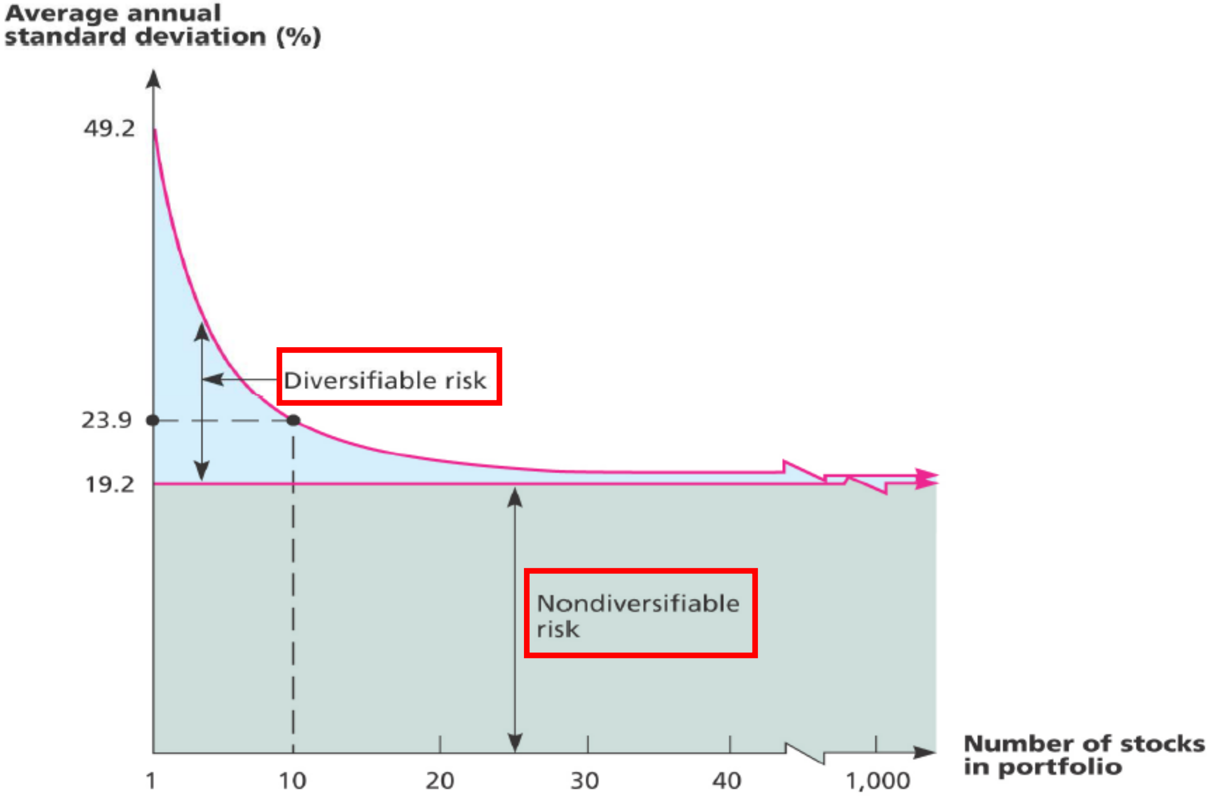
Fundamental truth of the investment world

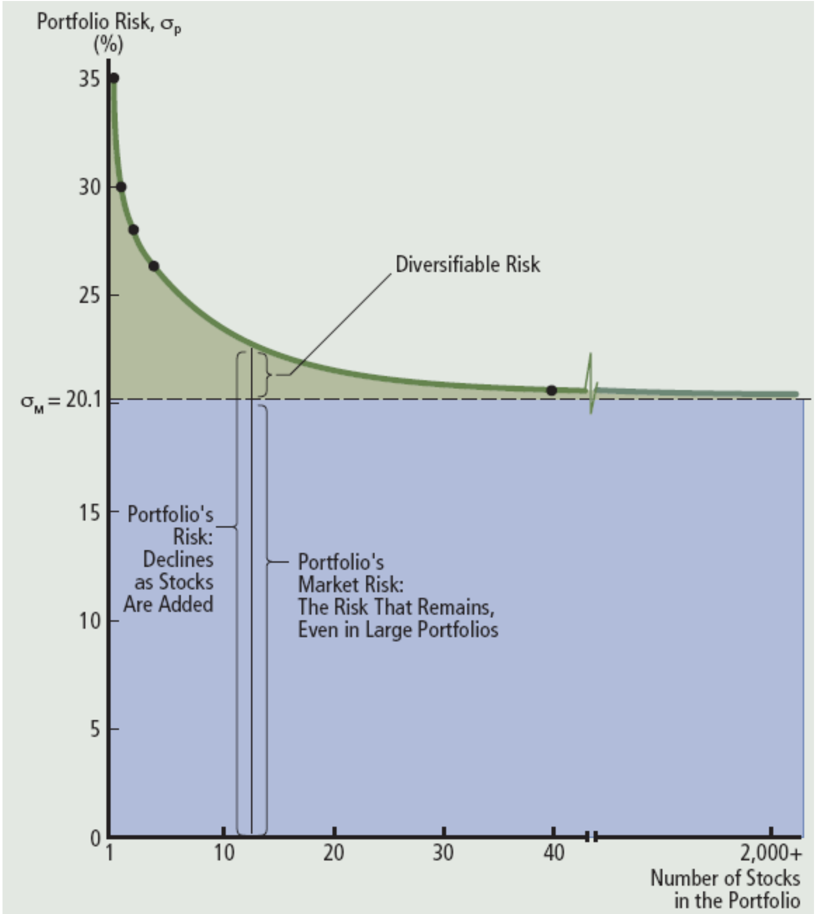
- The returns on securities tend to move up and down together
  - Not exactly together or proportionately

Events and Conditions Causing Movement in Returns

- Some things influence all stocks (market risk)
  - Political news, inflation, interest rates, war, *etc.*
- Some things influence only particular firms (business-specific risk)
  - Earnings reports, unexpected death of key executive, *etc.*
- Some things affect all companies within an industry
  - A labor dispute, shortage of a raw material

(1) Number of Stocks in Portfolio	(2) Average Standard Deviation of Annual Portfolio Returns	(3) Ratio of Portfolio Standard Deviation to Standard Deviation of a Single Stock
1	49.24%	1.00
2	37.36	.76
4	29.69	.60
6	26.64	.54
8	24.98	.51
10	23.93	.49
20	21.68	.44
30	20.87	.42
40	20.46	.42
50	20.20	.41
100	19.69	.40
200	19.42	.39
300	19.34	.39
400	19.29	.39
500	19.27	.39
1,000	19.21	.39


Average annual
standard deviation (%)
49.2
23.9
19.2
Diversifiable risk
Nondiversifiable
risk
1
10
20
30
40
1,000
Number of stocks
in portfolio


Portfolio Risk, σ_p (%)
35
30
25
σ_M = 20.1
15
10
5
0
1
10
20
30
40
2,000+
Number of Stocks in the Portfolio
Diversifiable Risk
Portfolio's Risk: Declines as Stocks Are Added
Portfolio's Market Risk: The Risk That Remains, Even in Large Portfolios

- Beta determines the volatility, or risk, of a security or fund in relation to that of its index or benchmark.
  - In contrast to standard deviation, which determines the volatility of a security or fund according to the disparity of its returns over a period of time.
- In the single factor Capital Asset Pricing Model, the index or benchmark is the “market” portfolio, often measured by the S&P 500 index.
- When beta is used to compare funds or to measure an investment manager’s performance, the benchmark is frequently the average of the funds in that mutual fund category.

- A fund with a beta very close to 1 means the fund's performance closely matches the index or benchmark.
  - A beta greater than 1 indicates greater volatility than the overall market.
  - A beta less than 1 indicates less volatility than the benchmark.
- Investors expecting the market to be bullish may choose funds exhibiting high betas, which increase investors' chances of earning high returns in up markets.
  - If an investor expects the market to be bearish in the near future, the funds that have betas less than 1 are a good choice because they would be expected to decline less in value than the index.
- Beta by itself is limited and can be skewed due to factors other than the market risk affecting the fund's volatility.

A stock's beta measures its market risk

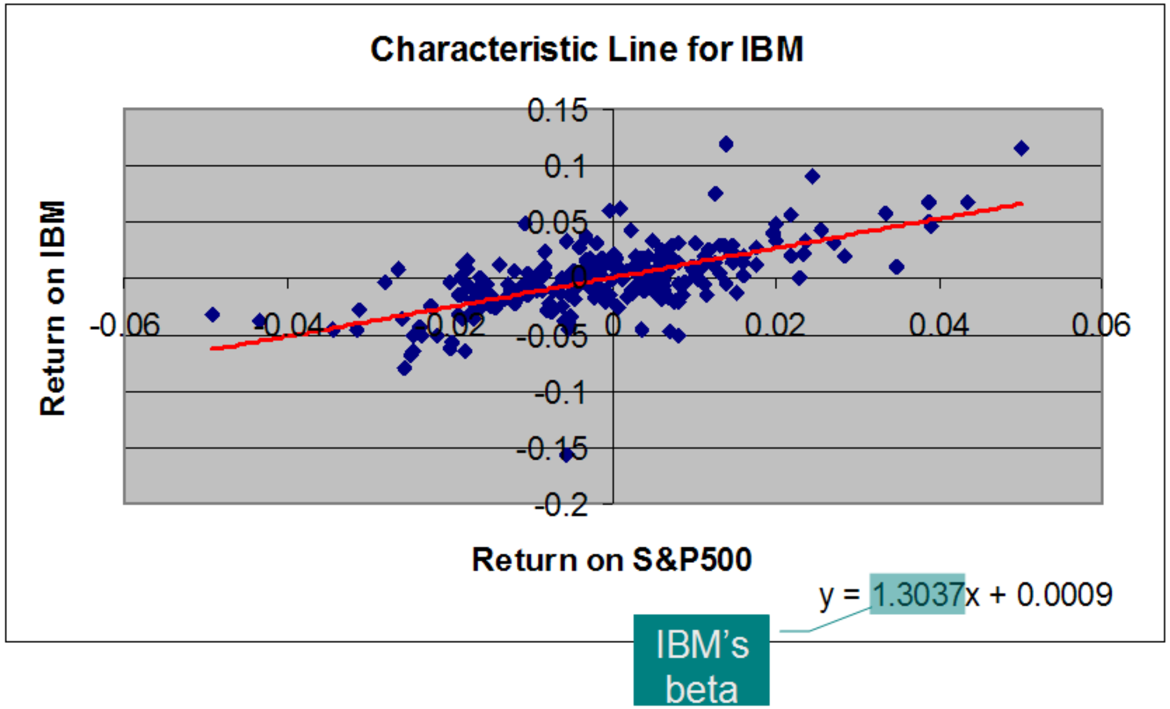
- It measures the variation of a stock's return which accompanies the market's variation in return

Developing Beta

- Beta is developed by determining the historical relationship between a stock's return and the return on a market index, such as the S&P500
  - The stock's characteristic line reflects the average relationship between its return and the market
    - Beta is the slope of the characteristic line

Projecting Returns with Beta

- Knowing a stock's beta enables us to estimate changes in its return given changes in the market's return
- Beta for a portfolio is the weighted average of the betas of the individual stocks within the portfolio


Characteristic Line for IBM
Return on IBM
Return on S&P500
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
y = 1.3037x + 0.0009
IBM's beta

Q: Conroy Corp. has a beta of 1.8 and is currently earning its owners a return of 14%. The stock market in general is reacting negatively to a new crisis in the Middle East that threatens world oil supplies. Experts estimate that the return on an average stock will drop from 12% to 8% because of investor concerns over the economic impact of a potential oil shortage as well as the threat of a limited war. Estimate the change in the return on Conroy shares and its new price.

A: Beta represents the past average change in Conroy's return relative to changes in the market's return.

$$b_{Conroy} = \frac{\Delta k_{Conroy}}{\Delta k_M} \text{ or } 1.8 = \frac{\Delta k_{Conroy}}{4\%}$$

$$\Delta k_{Conroy} = 7.2\%$$

The new return can be estimated as

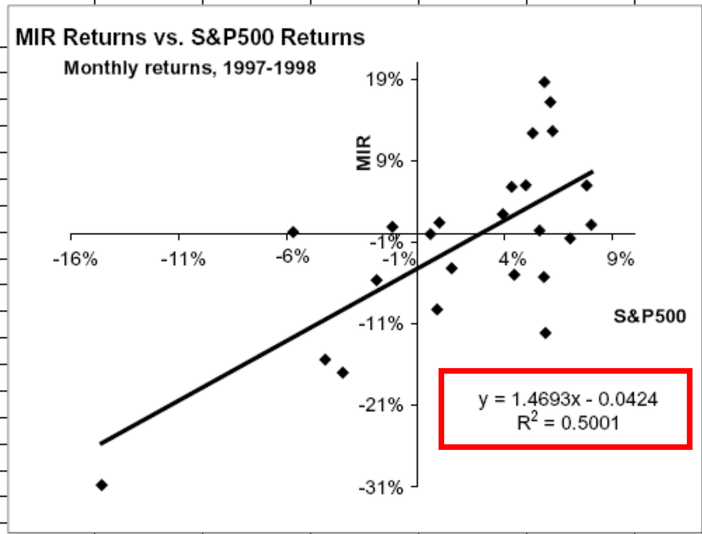
$$k_{Conroy} = 14\% - 7.2\% = 6.8\%$$

Sample of Betas & Their Standard Deviations

Company	Beta†	St. Deviation
AT&T	.76	24.2%
Bristol Myers Squibb	.81	19.8
Capital Holding	1.11	26.4
Digital Equipment	1.30	38.4
Exxon	.67	19.8
Ford Motor Co.	1.30	28.7
Genentech	1.40	51.8
McDonald's	1.02	21.7
McGraw-Hill	1.32	29.3
Tandem Computer	1.69	50.7

† based on 1984-89.

	A	B	C	D
1	SIMPLE REGRESSION EXAMPLE			
2	**Date**	**S&P 500 Index SPX**	**Mirage Resorts MIR**	
3	Jan-97	6.13%	16.18%	
4	Feb-97	0.59%	0.00%	
5	Mar-97	-4.26%	-15.42%	
6	Apr-97	5.84%	-5.29%	
7	May-97	5.86%	18.63%	
8	Jun-97	4.35%	5.76%	
9	Jul-97	7.81%	5.94%	
10	Aug-97	-5.75%	0.23%	
11	Sep-97	5.32%	12.35%	
12	Oct-97	-3.45%	-17.01%	
13	Nov-97	4.46%	-5.00%	
14	Dec-97	1.57%	-4.21%	
15	Jan-98	1.02%	1.37%	
16	Feb-98	7.04%	-0.54%	
17	Mar-98	4.99%	5.99%	
18	Apr-98	0.91%	-9.25%	
19	May-98	-1.88%	-5.67%	
20	Jun-98	3.94%	2.40%	
21	Jul-98	-1.16%	0.88%	
22	Aug-98	-14.58%	-30.81%	
23	Sep-98	6.24%	12.61%	
24	Oct-98	8.03%	1.12%	
25	Nov-98	5.91%	-12.18%	
26	Dec-98	5.64%	0.42%	
27				
28	Slope	1.4693	<-- =SLOPE(C3:C26,B3:B26)	
29		1.4693	<-- =COVAR(C3:C26,B3:B26)/VARP(B3:B26)	
30				
31	Intercept	-0.0424	<-- =INTERCEPT(C3:C26,B3:B26)	
32		-0.0424	<-- =AVERAGE(C3:C26)-B28*AVERAGE(B3:B26)	
33				
34	R-squared	0.5001	<-- =RSQ(C3:C26,B3:B26)	
35		0.5001	<-- =CORREL(C3:C26,B3:B26)^2	



What Does the Regression Mean?

The graph above shows the regression line as $y = 1.4693x - 0.0424$, $R^2 = 0.5001$. Since we're trying to understand the effect of the S&P 500 Index on MIR stock, we can attach the following meaning to the variables of the regression line:

- The "y" of the regression line stands for the monthly percentage return of MIR and the "x" stands for the monthly percentage return of the S&P 500 Index.
- The *slope* of the regression line is 1.4693. This tells us that, on average, a 1% increase in the S&P 500 monthly return caused a 1.4693% increase in the MIR monthly return. Of course, the reverse is true: On average a 1% decrease in the S&P 500 is related to a 1.4693% decrease in MIR's return.
- The fact that the slope of the regression is greater than 1 means that MIR is very sensitive to the S&P 500: Variations (increases or decreases) in the S&P 500 return cause larger variations in the MIR return. We return to this topic in Chapter 14.
- The *intercept* of the regression line is −0.0424. The intercept tells us that in months when the S&P 500 doesn't "move," MIR's return tends to decrease by 4.24%.
- The R^2 (pronounced "r squared") of the regression line says that 50.01% of the variability in the MIR returns is explained by the variability of the S&P 500 returns. This may seem sort of low but it's actually quite respectable: The R^2 of 50% says that half of MIR's return variability is explained by the variability of the S&P 500 Index. The other 50% of the return variability is presumably explained by factors that are unique to MIR. You wouldn't expect much more: If for some strange reason the R^2 were 100%, this would mean that *all* of MIR's returns are explained by the S&P 500 returns, which is clearly nonsense.